Int. J. Heat Mass Transfer. Vol. 19, pp. 61–69. Pergamon Press 1976. Printed in Great Britain

REGULAR EXPANSION SOLUTIONS FOR HEAT OR MASS TRANSFER IN CONCENTRATED TWO-PHASE PARTICULATE SYSTEMS AT SMALL PECLET AND REYNOLDS NUMBERS

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(*Received* 23 *May* 1974 *and in revised form* 18 *March* 1975)

Abstract—General solutions for small Peclet number heat or mass transfer in concentrated two-phase particulate systems submerged in arbitrary Stokesian flows with arbitrary temperature or concentration boundary conditions were obtained using regular perturbation expansions of the dependent variable in powers of the Peclet number. Uniformly valid solutions for all orders of approximation were obtained within the domain of interest, subject to a restriction imposed on the value of the Peclet number by the dimensions of that domain. The solutions were specialized to cases of practical interest. For a particulate system submerged in a uniform flow with a linear temperature gradient in the direction of flow, the results reduced to those of Maxwell for effective electrical conductivity of a composite medium. Specific relations were also obtained for local and overall interfacial fluxes and Nusselt numbers. The results reflect the effects of particle volume fraction and retardation of internal circulation by surfactant impurities.

NOMENCLATURE

a,	radius of typical particle;
c,	concentration;
D,	molecular (binary) diffusivity;
G,	rate of shear;
$G(\eta, \eta')$,	spherical Green's function defined in equation (21);
$\mathbf{g}$,	acceleration due to gravity;
$g_m(\eta, \eta')$,	radial component of spherical Green's function, defined in equation (22);
$\mathbf{J}$,	interfacial flux;
k,	thermal conductivity of homogeneous phase;
k_{eff},	effective thermal conductivity of two-phase system;
m,	distribution coefficient;
P_n^p,	associated Legendre's function of degree n, order p;
Q,	overall rate of interfacial heat transfer;
r,	radial distance;
S_n,	solid surface harmonic of degree n;
T,	temperature;
$\mathbf{t}$,	unit vector in spherical coordinates system;
U,	uniform reference velocity;
U_s,	terminal sedimentation velocity;
V,	volume;
$\mathbf{v}$,	nondimensional velocity vector.

Greek symbols

α,	thermal diffusivity;
β,	viscosity ratio;
γ,	reciprocal nondimensional radius of spherical cell;
$\tilde{\gamma}$,	"interfacial retardation viscosity" defined in [4];
ζ,	thermal or molecular diffusivity ($\zeta \equiv \alpha$ or $\zeta \equiv D$);
η,	nondimensional radial distance;
θ,	cone angle;
μ,	viscosity;
ξ,	thermal conductivity, or molecular diffusivity;
ρ,	density;
Φ,	volume fraction of dispersed phase;
ϕ,	azimuthal angle;
ψ,	dependent variable ($\psi \equiv T$ or $\psi \equiv c$).

Subscripts

r,	radial;
θ,	cone;
ϕ,	azimuthal;
1, 2,	values of dependent variable at antipodal points of spherical cell;
$<, >$,	in equation (22), the smaller, greater value between η and η'.

Superscripts

α,	indicates phase in general;
c,	pertains to continuous phase;
d,	pertains to dispersed phase;
0,	pertains to unperturbed field conditions.

Special signs

$\tilde{\nabla}$,	nondimensional differential operator defined in equation (A7);
$-$,	average quantity.

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INTRODUCTION

IN PREVIOUS publications [15, 16] it has been proposed, that problems of low Peclet number heat or mass transfer in concentrated two-phase particulate systems with dominant resistance to interfacial transfer in the continuous phase, can be solved by regular perturbation expansions of the temperature or concentration in powers of the Peclet numbers. The spherical cell, of non-dimensional radius γ^{-1}, was employed to statistically represent a multiparticle assemblage [3], and uniformly valid solutions were obtained for all orders of approximation within the domain of interest $1 < \eta < \gamma^{-1}$. The solutions obtained were subject to the restrictive relationship $Pe \ll \gamma < 1$.

In the present work the previous analyses are generalized to multiparticle assemblages submerged in arbitrary Stokesian flows and subjected to boundary conditions of the most general nature. Furthermore, the analyses are extended to systems with resistances to transfer in both phases of comparable magnitude. The general solutions obtained reflect the effects on the transfer processes of the volume fraction of the dispersed phase Φ and of adventitious surfactant impurities, adsorbed at the particles' interfaces. The solutions are then specialized to cases of specific interest, and the results are employed to calculate rates of interfacial transfer, and average bulk properties, e.g. effective thermal conductivity, of multiparticle systems.

It should be emphasized here, that in view of the statistical nature of the spherical cell model employed in the present analysis to represent the multiparticle assemblage, all the results predicted by the present theory should be regarded as ensemble-averaged expected values.

THEORY

Statement of the Problem

Consider a uniformly-sized assemblage of rigid or deformable particles approximately spherical in shape. The assemblage is submerged in a steady arbitrary Stokesian flow and is placed within an arbitrary temperature or concentration field (here termed the unperturbed field). The assemblage is substituted in the statistical sense by a spherical cell of the continuous phase material enclosing a typical particle of the dispersed phase.

With the simplifying assumptions of: (a) Steady transport processes; (b) Slow viscous motion of incompressible, homogeneous, isotropic, Newtonian fluids in each phase; (c) Constancy of physical properties; (d) Absence of sources and sinks; (e) Negligible viscous dissipation, compressive work, radiative heat transfer, and natural convection; (f) Negligible thermodynamic coupling [17]; and (g) Negligible transfer-induced interfacial fluxes [4]; the conservation of energy and chemical species equations for each homogeneous, isotropic phase α are unified into

$$Pe^{\alpha}\mathbf{v}^{\alpha}\,.\,\nabla\psi^{\alpha} = \nabla^{2}\psi^{\alpha}. \tag{1}$$

where in general

$$Pe^{\alpha} \equiv \bar{G}a^{2}/\zeta^{\alpha}. \tag{2}$$

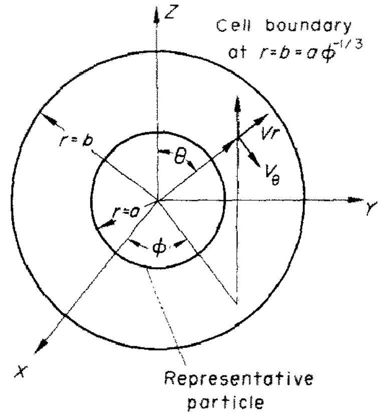

FIG. 1. Geometry of spherical cell and coordinate system.

In equation (1) $\psi = T$ for heat transfer and $\psi = c$ for mass transfer. Accordingly, in equation (2) $\zeta = \alpha$ or $\zeta = D$ for heat or mass transfer, respectively.

The solutions of equation (1) must satisfy the following boundary condition, consistent with the statistical representation of a multiparticle system by the cell model (see Fig. 1 for geometry of the system).

$$\psi^{c} = \psi^{0}(\theta,\phi) \quad \text{at} \quad \eta = \gamma^{-1}. \tag{3}$$

They also must satisfy the following set of interfacial matching conditions

$$\left.\begin{aligned} &\psi^{c} = m\psi^{d}, &&(4)\\ &\xi^{c}\frac{\partial\psi^{c}}{\partial\eta} = \xi^{d}\frac{\partial\psi^{d}}{\partial\eta}, &&(5)\end{aligned}\right\} \quad \text{at} \quad \eta = 1,$$

where $\xi = k$ or $\xi = D$ for heat or mass transfer, respectively. Note, that the distribution coefficient $m = 1$ for heat transfer.

Furthermore, it is required that there be no singularities associated with the solutions for ψ^{d} at the origin.

Linearization Process

For small Peclet numbers, i.e. $Pe^{\alpha} \ll 1$, the solutions of equation (1) may be obtained by means of a regular perturbation expansion of ψ^{α} in terms of powers of Pe^{α}, namely

$$\psi^{\alpha} = \sum_{j=0}^{\infty} (Pe^{\alpha})^{j}\psi^{\alpha}_{(j)}, \tag{6}$$

whereby the following set of linearized equations results

$$\mathbf{v}^{\alpha}\,.\,\nabla\psi^{\alpha}_{(j-1)} = \nabla\,.\,(\mathbf{v}^{\alpha}\psi^{\alpha}_{(j-1)}) = \nabla^{2}\psi^{\alpha}_{(j)}. \tag{7}$$

Application of the expansions (6) to the boundary and the interfacial matching conditions (3)–(5) results in

$$\left.\begin{aligned} &\psi^{c}_{(0)} = \psi^{0}, &&(8)\\ &\psi^{c}_{(1)} = \psi^{c}_{(2)} = \ldots = \psi^{c}_{(\infty)} = 0, &&(9)\end{aligned}\right\} \quad \text{at} \quad \eta = \gamma^{-1},$$

$$\left.\begin{aligned} &\psi^{c}_{(j)} = \delta_{j}\psi^{d}_{(j)}, &&(10)\\ &\frac{\partial\psi^{c}_{(j)}}{\partial\eta} = \varepsilon_{j}\frac{\partial\psi^{d}_{(j)}}{\partial\eta}, &&(11)\end{aligned}\right\} \quad \text{at} \quad \eta = 1.$$

Here $\delta_{j} \equiv m(\zeta^{c}/\zeta^{d})^{j}$,

and $$\varepsilon_{j} \equiv \frac{\xi^{d}}{\xi^{c}}\left(\frac{\zeta^{c}}{\zeta^{d}}\right)^{j}.$$

It is apparent, that the requirement of absence of singularities at the origin applies to all $\psi^d_{(j)}$.

Zero Order Solutions

The zero order solutions are treated separately from the higher order ones in view of the inhomogeneous boundary condition (8). The solutions of the zero order Laplace equations

$$\nabla^2\psi^\alpha_{(0)} = 0, \tag{12}$$

in spherical coordinates are sums of solid spherical harmonics, which are expanded in terms of solid surface harmonics in the form

$$\psi^\alpha_{(0)} = \sum_{k=0}^{\infty} (J^\alpha_{(0)_k}\eta^k + J^\alpha_{(0)_{-k-1}}\eta^{-k-1})S_k(\theta, \phi). \tag{13}$$

Assuming that the unperturbed field can also be represented in the form

$$\psi^0 = \sum_{k=0}^{\infty} (J^0_k\eta^k + J^0_{-k-1}\eta^{-k-1})S_k(\theta, \phi), \tag{14}$$

applying the boundary (8) and the interfacial matching conditions (10) and (11), restricting the spherical harmonics in the solution for the dispersed phase to positive degree only in view of the requirement for the lack of singularities at the origin, utilizing the orthogonality properties of solid surface harmonics, and solving the resulting system of algebraic equations for the unknown coefficients $J^c_{(0)_k}$, $J^c_{(0)_{-k-1}}$ and $J^d_{(0)_k}$ in terms of the known, but yet arbitrary coefficients J^0_k and J^0_{-k-1} of the unperturbed field, one obtains

$$J^c_{(0)_k} = \frac{(J^0_k\gamma^{-1} + J^0_{-k-1}\gamma^{k+1})\left[\left(\frac{k+1}{k}\right)\frac{\xi^c}{\xi^d} + \frac{1}{m}\right]}{\left[\left(\frac{k+1}{k}\right)\frac{\xi^c}{\xi^d} + \frac{1}{m}\right]\gamma^{-k} + \left(\frac{\xi^c}{\xi^d} - \frac{1}{m}\right)\gamma^{k+1}}, \tag{15}$$

$$J^c_{(0)_{-k-1}} = \frac{(J^0_k\gamma^{-k} + J^0_{-k-1}\gamma^{k+1})\left(\frac{\xi^c}{\xi^d} - \frac{1}{m}\right)}{\left[\left(\frac{k+1}{k}\right)\frac{\xi^c}{\xi^d} + \frac{1}{m}\right]\gamma^{-k} + \left(\frac{\xi^c}{\xi^d} - \frac{1}{m}\right)\gamma^{k+1}}, \tag{16}$$

and

$$J^d_{(0)_k} = \frac{1}{m}\,\frac{(J^0_k\gamma^{-k} + J^0_{-k-1}\gamma^{k+1})\frac{\xi^c}{\xi^d}\left(\frac{2k+1}{k}\right)}{\left[\left(\frac{k+1}{k}\right)\frac{\xi^c}{\xi^d} + \frac{1}{m}\right]\gamma^{-k} + \left(\frac{\xi^c}{\xi^d} - \frac{1}{m}\right)\gamma^{k+1}}. \tag{17}$$

General Solutions for Higher Orders of Approximation

A formal scheme of solutions is now presented for the higher orders of approximation. For any jth order equations

$$\nabla\cdot(\mathbf{v}^\alpha\psi^\alpha_{(j-1)}) = \nabla^2\psi^\alpha_{(j)}, \tag{18}$$

the left-hand side $\mathbf{v}^\alpha$ and $\psi^\alpha_{(j-1)}$ are known functions, such that $\nabla\cdot(\mathbf{v}^\alpha\psi^\alpha_{(j-1)}) \equiv F^\alpha_{(j)}$ are also known functions. Then, equation (18) will have general solutions, composed of homogeneous parts, resulting from the homogeneous Laplace equation $\nabla^2\psi^\alpha_{(j)_H} = 0$, of the form

$$\psi^\alpha_{(j)_H} = \sum_{q=0}^{s+m} (J^\alpha_{(j)_q}\eta^q + J^\alpha_{(j)_{-q-1}}\eta^{-q-1})S_q(\theta, \phi), \tag{19}$$

and particular solutions of the Poisson-type equations $\nabla^2\psi^\alpha_{(j)_p} = F^\alpha_{(j)}$,

$$\psi^\alpha_{(j)_p} = \frac{1}{4\pi}\int_{V'} F^\alpha_{(j)}(\eta')G^\alpha(\boldsymbol{\eta}, \boldsymbol{\eta}')\,\mathrm{d}V'. \tag{20}$$

In solutions (20) the spherical Green function in spherical coordinates is given in Jackson's [7] notation by

$$G^\alpha(\boldsymbol{\eta}, \boldsymbol{\eta}') = \sum_{m=0}^{\infty} g^\alpha_m(\eta, \eta')S_m(\theta, \phi)S_m(\theta', \phi'), \tag{21}$$

and

$$g^c_m(\eta, \eta') = \frac{1}{1+\gamma^{2m+1}}\left(\eta^m_< - \frac{1}{\eta^{m+1}_<}\right)\left(\frac{1}{\eta^{m+1}_>} - \gamma^{2m+1}\eta^m_>\right). \tag{22}$$

The particular solution can in general be represented in the form

$$\psi^\alpha_{(j)_p} = \sum_{q=0}^{s+m} X^\alpha_{(j)_q}(\eta)S_q(\theta, \phi). \tag{23}$$

Applying the linearized boundary and interfacial matching conditions (9)–(11), utilizing again the orthogonality properties of solid spherical harmonics, and solving the resulting set of algebraic equations, one finds

$$J^c_{(j)_q} = \frac{V_{(j)_q}\delta_j\gamma^{2q+1} - U_{(j)_q}\varepsilon_j\gamma^{2q+1} + T_{(j)_q}\gamma^q(q\delta_j + q\varepsilon_j + \delta_j)}{q(\delta_j - \varepsilon_j)(\gamma^{2q+1} - 1) + \delta_j(2q+1)}, \tag{24}$$

$$J^c_{(j)_{-q-1}} = \frac{T_{(j)_q}q\gamma^q(\delta_j - \varepsilon_j) + U_{(j)_q}q\varepsilon_j - V_{(j)_q}\delta_j}{q(\delta_j - \varepsilon_j)(\gamma^{2q+1} - 1) + \delta_j(2q+1)}, \tag{25}$$

and

$$J^d_{(j)_q} = \frac{(V_{(j)_q} - qU_{(j)_q})(\gamma^{2q+1} - 1) + (T_{(j)_q}\gamma^q - U_{(j)_q})(2q+1)}{q(\delta_j - \varepsilon_j)(\gamma^{2q+1} - 1) + \delta_j(2q+1)}, \tag{26}$$

where the following functions were defined;

$$T_{(j)_q} \equiv -X^c_{(j)_q}(\gamma^{-1}),$$
$$U_{(j)_q} \equiv \delta_j X^d_{(j)_q}(1) - X^c_{(j)_q}(1),$$

and

$$V_{(j)_q} \equiv \frac{\partial}{\partial\eta}[\varepsilon_j X^d_{(j)_q}(1) - X^c_{(j)_q}(1)].$$

The derivations of expressions for $X^\alpha_{(j)_q}(\eta)$ are presented in Appendix A. This completes the general solutions for the higher orders of approximation.

Illustrative Examples

1. *Uniform unperturbed field*

For the case of a uniform unperturbed field, in equation (13) $k = 0$, $S_0(\theta, \phi) = 1$, and $J^0_{-1} = 0$, whereby $\psi^0 = J^0_0$. Then, from equations (15)–(17) one finds that, as long as $\xi^c/\xi^d \neq 0$ and m is finite, $J^c_{(0)_0} = J^0_0$, $J^c_{(0)_{-1}} = 0$ and $J^d_{(0)_0} = (1/m)J^0_0$. This indicates, that the temperatures or concentrations in each phase are uniform, that both contracting phases are in complete equilibrium, and that, accordingly, no interfacial transfer takes place.

2. *Linear gradient of the unperturbed field*

(a) *Uniform flow.* Two different orientations of the linear unperturbed field gradient with respect to the direction of the uniform flow are considered.

(i) Linear gradient co- or countercurrent to the uniform flow. When the linear temperature or concentration gradient of the unperturbed field is directed parallel to the axis $\theta = 0$, $\theta = \pi$, in equation (17) $k = 0$ and $k = 1$, with $S_1(\theta, \phi) = \cos\theta$. Then,

$$\psi^0 = \tfrac{1}{2}[(\psi_2 - \psi_1)\cos\theta + (\psi_2 + \psi_1)]. \tag{27}$$

To order $(Pe^\alpha)^2$ the distributions ψ^c and ψ^d are then obtained from equations (6), (15)–(17), (19), (23)–(26), (A2) and (A3), whereby

$$\begin{aligned}\psi^c = {} & (J^c_{(0)_1}\eta + J^c_{(0)_{-2}}\eta^{-2})\cos\theta + J^c_{(0)_0} \\ & \pm Pe^c[(J^c_{(1)_2}\eta^2 + J^c_{(1)_{-3}}\eta^{-3} + X^c_{(1)_2})(\tfrac{3}{2}\cos^2\theta - \tfrac{1}{2}) \\ & + J^c_{(1)_0} + J^c_{(1)_{-1}}\eta^{-1} + X^c_{(1)_0}] + O(Pe^c)^2,\end{aligned} \tag{28}$$

and

$$\begin{aligned}\psi^d = {} & J^d_{(0)_1}\eta\cos\theta + J^d_{(0)_0} \\ & + Pe^d[(J^d_{(1)_2}\eta^2 + X^d_{(1)_2})(\tfrac{3}{2}\cos^2\theta - \tfrac{1}{2}) \\ & + J^d_{(1)_0} + X^d_{(1)_0}] + O(Pe^d)^2.\end{aligned} \tag{29}$$

In solutions (28) and (29) the first order terms should be taken with the positive sign if the linear gradient is cocurrent, and with a negative sign if countercurrent to the flow, respectively. Derivations leading to solutions (28) and (29) and the functions $J^c_{(0)_1}$, $J^c_{(0)_{-2}}, \ldots$, etc., are listed in Appendix B.

(ii) Linear gradient normal to the direction of uniform flow. When the linear temperature or concentration gradient of the unperturbed field is normal to the axis $\theta = 0$, $\theta = \pi$, the distributions ψ^c and ψ^d are to order $(Pe^\alpha)^2$ given by

$$\begin{aligned}\psi^c = {} & (J^c_{(0)_1}\eta + J^c_{(0)_{-2}}\eta^{-2})\sin\theta\sin\phi + J^c_{(0)_0} \\ & + Pe^c\tfrac{1}{2}(J^c_{(1)_2}\eta^2 + J^c_{(1)_{-3}}\eta^{-3} + X^c_{(1)_2}) \\ & \times 3\sin\theta\cos\theta\sin\phi + O(Pe^c)^2,\end{aligned} \tag{30}$$

and

$$\begin{aligned}\psi^d = {} & J^d_{(0)_1}\eta\sin\theta\sin\phi + J^d_{(0)_0} + Pe^d\tfrac{1}{2}(J^d_{(1)_2}\eta^2 + X^d_{(1)_2}) \\ & \times 3\sin\theta\cos\theta\sin\phi + O(Pe^d)^2.\end{aligned} \tag{31}$$

In equations (30) and (31) the functions $J^c_{(0)_1}$, $J^c_{(0)_{-2}}$, etc. are identical to those in equations (28) and (29).

(b) *Homogeneous shear flow.* With the linear temperature or concentration gradient of the unperturbed field oriented co- or countercurrently to the direction of the flow, the distributions of ψ^c and ψ^d to order $(Pe^\alpha)^2$ are now

$$\begin{aligned}\psi^c = {} & (J^c_{(0)_1}\eta + J^c_{(0)_{-2}}\eta^{-2})\cos\theta + J^c_{(0)_0} \\ & \pm Pe^c(J^c_{(1)_3}\eta^3 + J^c_{(1)_{-4}}\eta^{-4} + X^c_{(1)_3}) \\ & \times 15\sin 2\phi\sin^2\theta\cos\theta + O(Pe^c)^2,\end{aligned} \tag{32}$$

and

$$\begin{aligned}\psi^d = {} & J^d_{(0)_1}\eta\cos\theta + J^d_{(0)_0} \pm Pe^d(J^d_{(1)_3}\eta^3 + X^d_{(1)_3}) \\ & \times 15\sin 2\phi\sin^2\theta\cos\theta + O(Pe^d)^2.\end{aligned} \tag{33}$$

In the above equations $J^c_{(0)_1}$, $J^c_{(0)_{-2}}$, $J^c_{(0)_0}$, $J^d_{(0)_1}$ and $J^d_{(0)_0}$ again have the same significance as before. The functions $J^c_{(1)_3}$, $J^c_{(1)_4}$, $J^d_{(1)_3}$, $X^c_{(1)_3}$ and $X^d_{(1)_3}$ are listed in Appendix B.

Effective Thermal Conductivity

It is now possible to employ the various distributions, given by equations (28)–(33) to evaluate average bulk properties. Restricting the treatment to heat transfer, the bulk property of effective thermal conductivity of the multiparticle system is calculated for all the illustrative cases from the standard operational definition as the ratio of the heat flux in a given direction to the mean temperature gradient in that direction. Thus

$$k_{\text{eff}} = \frac{2Q}{\Delta T \pi a \gamma^{-1}}. \tag{34}$$

The overall rate of heat transferred in the co- or countercurrent situations is found in a manner, similar to that employed for determination of the average relative velocity between the phases [13], namely

$$\begin{aligned}Q = {} & -\int_0^{2\pi}\int_1^{\gamma^{-1}} k^c\left(\frac{\partial T^c}{\partial\theta}\right)_{\theta=\pi/2} a\,d\eta\,d\phi \\ & -\int_0^{2\pi}\int_0^{1} k^d\left(\frac{\partial T^d}{\partial\theta}\right)_{\theta=\pi/2} a\,d\eta\,d\phi.\end{aligned} \tag{35}$$

Introducing the values of T^c and T^d from distributions (28) and (29), and (32) and (33), it is found, that to order $(Pe^\alpha)^2$ in both cases

$$\frac{k_{\text{eff}}}{k^d} = \frac{k^c\left[\left(2\dfrac{k^c}{k^d}+1\right)(\gamma^{-2}-1) - 2\left(\dfrac{k^c}{k^d}-1\right)(\gamma-1)+3\right]}{k^d\gamma^{-1}\left[\left(2\dfrac{k^c}{k^d}+1\right)\gamma^{-1} + \left(\dfrac{k^c}{k^d}-1\right)\gamma^2\right]}. \tag{36}$$

Expression (36) can be readily recognized as Maxwell's [10] result for the effective electrical conductivity of a composite medium. It is not too difficult to verify, that an identical result is obtained from distributions (30) and (31) for the case of a linear temperature gradient normal to the direction of the uniform flow. It is interesting to note, that the first order convective terms do not contribute in these cases to the effective thermal conductivity.

Interfacial Transfer

The local interfacial fluxes are defined by

$$(\mathbf{J})_{\eta=1} = -\zeta^\alpha\left(\frac{\partial\psi^\alpha}{\partial\eta}\right)_{\eta=1}. \tag{37}$$

For the case of a linear gradient of the unperturbed field co- or countercurrent to the direction of the uniform flow, the surface distribution of interfacial fluxes is then, from equations (28) and (29)

$$\begin{aligned}(\mathbf{J})_{\eta=1} = {} & \\ & -\zeta^d\left(J^d_{(0)_1}\cos\theta \pm Pe^d\left\{\left[2J^d_{(1)_2} + \frac{\partial}{\partial\eta}(X^d_{(1)_2})_{\eta=1}\right]\right.\right. \\ & \left.\left.\times(\tfrac{3}{2}\cos^2\theta - \tfrac{1}{2}) + \frac{\partial}{\partial\eta}(X^d_{(1)_0})_{\eta=1}\right\}\right) + O(Pe^d)^2.\end{aligned} \tag{38}$$

Similarly, for a linear gradient normal to the direction of the uniform flow

$$(\mathbf{J})_{\eta=1} = -\zeta^d \left\{ J^d_{(0)_1} \sin\theta \sin\phi + Pe^d \left[2J^d_{(1)_2} + \frac{\partial}{\partial\eta}(X^d_{(1)_2})_{\eta=1} \right] \times \tfrac{3}{2} \sin\theta\cos\theta\sin\phi \right\} + O(Pe^d)^2. \quad (39)$$

Finally, when the linear gradient is co- or countercurrent to the direction of the homogeneous shear (Couette) flow

$$(\mathbf{J})_{\eta=1} = -\zeta^d \left\{ J^d_{(0)_1} \cos\theta \pm Pe^d \left[3J^d_{(1)_3} + \frac{\partial}{\partial\eta}(X^d_{(1)_3})_{\eta=1} \right] \times 15 \sin 2\phi \sin^2\theta \cos\theta \right\} + O(Pe^d)^2. \quad (40)$$

It is interesting to observe, that the average interfacial fluxes defined by

$$(\mathbf{J})_{\eta=1} = \frac{1}{2\pi a^2} \int_0^{2\pi} \int_0^{\pi} (\mathbf{J})_{\eta=1} a^2 \sin\theta \,\mathrm{d}\theta \,\mathrm{d}\phi,$$

and the average Nusselt numbers for interfacial transfer, defined by

$$\overline{Nu} = (\bar{\mathbf{J}})_{\eta=1} / (\zeta^z \overline{\nabla \psi^z}),$$

are found to order $(Pe^z)^2$ for all the above cases to be equal to zero.

DISCUSSION

Linearization of the unified conservation of energy and chemical species equation by a regular perturbation expansion of the dependent variables in terms of power series of the small Peclet numbers permitted to obtain general solutions for concentrated two-phase particulate systems, submerged in arbitrary, Stokesian flows within arbitrary temperature on concentration unperturbed fields. Convergence of the solutions for any order of approximation was achieved by satisfying the nonhomogeneous boundary condition in the zero order of approximation solutions, and subjecting higher order solutions to homogeneous boundary conditions. The general temperature or concentration distributions derived are uniformly valid within the domain of the spherical cell employed to statistically represent the multiparticle assemblage, subject to the restrictive relationship $Pe \ll \gamma < 1$. The temperature or concentration distribution, and other quantities derived thereof should, of course, be viewed as average expected quantities, characteristic, in the statistical sense, of the entire system, rather than actual distributions, likely to be encountered around any particular particle.

The general solutions were first specialized to a system, placed within a uniform unperturbed field. It was shown, that, unless it be assumed that the conductivity or diffusivity of the dispersed phase is infinite, there can be no interfacial transfer between the phases. Since this conclusion involves the leading term of pure radial conduction, it applies equally well to the case of a single particle submerged in an infinite, unbounded field, with uniform conditions at infinity. Hence it seems that those numerous analyses (e.g. [1, 8] etc.) which have the case of pure radial conduction from an isothermal particle as a starting point for low Peclet number expansions of one kind or another, are in essence physically unrealistic.

The specialization of the general solutions next to cases, in which the multiparticle system was placed in an unperturbed field with a linear gradient, showed that the leading term for zero Peclet number is identical with that given by Maxwell for electric conductivity of composite medium. Maxwell claimed that his results apply strictly to dilute systems, in which interaction effects between particles are not taken into account. However it has been shown since in numerous investigations (e.g. [2, 5, 11] etc.) that experimental data on effective thermal conductivity in concentrated systems are also adequately correlated by Maxwell's formula. As a point of interest, in Table 1 the recent data of Singh *et al.* [12] for the effective thermal conductivity of metal wicks, saturated with water, is compared with values, predicted from equation (36). The satisfactory agreement between the experimental data, and theory, particularly in the range of higher concentrations is noteworthy.

Table 1. Effective thermal conductivity of water saturated wicks, data of Singh *et al.* [12]

		K_{eff}/k^c	
Wick	Porosity (%)	Experimental	Calculated, equation (36)
Ni 200	81·5	0·11	0·131
Ni 200	43·9	0·45	0·463
Ni 200	33·4	0·506	0·575
Ni 200	28·8	0·577	0·63
430 SS	75·9	0·20	0·202
430 SS	82·6	0·16	0·125

In the expressions derived for effective thermal conductivity and surface-averaged interfacial transfer, in the cases considered, first-order effects due to convection are absent. However, it is evident from previous investigations of cases with dominant resistance in the continuous phase [15], that second-order terms do appear. Accordingly, the expressions will differ from case to case. Their use in situations, other than those for which they have been derived, is, therefore, unwarranted, even on an ad hoc basis. It is conceivable, that in other cases, not considered here, even first-order convective effects may appear, which may make indiscriminate use of values, based on the zero-order pure conduction case, even more erroneous.

Acknowledgement—This work was made possible through NASA Grant No. NGL-36-003-064 and in part through a grant from the Yad Avi Hayishuv Foundation.

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APPENDIX A

In the linearized equations (7) the dimensional velocity vector $\mathbf{V}^{\alpha}$ is given by Lamb's [9] general solution of the steady creeping flow equations in spherical coordinates

$$\mathbf{V}^{\alpha} = \sum_{n=-\infty}^{\infty}\left[\nabla x(\mathbf{r}\chi_n^{\alpha}) + \nabla\phi_n^{\alpha} + \frac{(n+3)}{2\mu^{\alpha}(n+1)(2n+3)}r^2\nabla P_n^{\alpha} - \frac{n}{\mu^{\alpha}(n+1)(2n+3)}\mathbf{r}P_n^{\alpha}\right]. \tag{A1}$$

Recall that the solid spherical harmonics are expanded in terms of solid surface harmonics in the following manner:

$$P_n^d = \mu^d\eta^n a^{-1}A_nS_n(\theta,\phi),\quad P_n^c = \mu^c\eta^n a^{-1}D_nS_n(\theta,\phi),$$
$$P_{-n-1}^c = \mu^c\eta^{-n-1}a^{-1}G_nS_n(\theta,\phi),\quad \phi_n^d = \eta^n aB_nS_n(\theta,\phi),$$
$$\phi_n^c = \eta^n aE_nS_n(\theta,\phi),\quad \phi_{-n-1}^c = \eta^{-n-1}aH_nS_n(\theta,\phi),$$
$$\chi_n^d = \eta^n C_nS_n(\theta,\phi),\quad \chi_n^c = \eta^n F_nS_n(\theta,\phi),$$
$$\chi_{-n-1}^c = \eta^{-n-1}I_nS_n(\theta,\phi).$$

In these expansions, the coefficients $A_n, B_n, \ldots, I_n$ are expressed in terms of coefficients $\alpha_n, \beta_n, \ldots, \gamma_{-n-1}$ of a known, although as yet arbitrary flow field. For lack of space, these coefficients are not listed here, but may be found in [14]. It should be noted, that the products A_nS_n, $B_nS_n, \ldots$, represent expansions of the general forms

$$A_nS_n(\theta,\phi) = \sum_{p=0}^{n}(A_n^p\cos p\phi + \hat{A}_n^p\sin p\phi)P_n^p(\sin\theta),$$

Using now expression (A1) for the velocity vector and after some rather lengthy computations it is found, that

$$\begin{aligned}X_{(j)_q}^d(\eta) = {}&\frac{1}{4\pi\bar{G}a^2}\sum_l\sum_n\sum_m g_q^{s,m}\int_0^1\int_0^{2\pi}\int_0^{\pi}\left[C_n\sum_{s=0}^{l+n-1}f_s^{n,l}[J_{(j-1)_l}^d\eta'^{l+n-1} + X_{(j-1)_l}^d(\eta')\eta'^{n-1}]\right.\\ &\frac{A_n}{2(2n+3)}\sum_{s=0}^{l+n}\left(g_s^{n,l}\left\{J_{(j-1)_l}^d[(n+l+3)\eta'^{n+l} - 2(n+l+1)\eta'^{n+l-2}] + \frac{1}{\eta'^2}\frac{\partial}{\partial\eta'}[X_{(j-1)_l}^d(\eta')(\eta'^{n+3} - 2\eta'^{n+1})]\right\}\right.\\ &\left.\left.+ h_s^{n,l}\frac{1}{(n+1)}\{J_{(j-1)_l}^d[\eta(n+3)\eta'^{n+l} - 2(n+1)\eta'^{n+l-2}] + X_{(j-1)_l}^d(\eta')[n(n+3)\eta'^n - 2(n+1)\eta'^{n-2}]\}\right)\right]\\ &\times g_m^d(\eta,\eta')S_m(\theta',\phi')\eta'^2\sin\theta'\,\mathrm{d}\theta'\,\mathrm{d}\phi'\,\mathrm{d}\eta'.\end{aligned} \tag{A2}$$

$$\begin{aligned}X_{(j)_q}^c(\eta) = {}&\frac{1}{4\pi\bar{G}a^2}\sum_l\sum_n\sum_m g_q^{s,m}\int_1^{\gamma^{-1}}\int_0^{2\pi}\int_0^{\pi}\left[\sum_{s=0}^{l+n-1}f_s^{n,l}\left\{F_n[J_{(j-1)_l}^c\eta'^{l+n-1} + J_{(j-1)_{-l-1}}^c\eta'^{n-l-2} + X_{(j-1)_l}^c(\eta')\eta'^{n-1}]\right.\right.\\ &\left.+ I_n[J_{(j-1)_l}^c\eta'^{l-n-2} + J_{(j-1)_{-l-1}}^c\eta'^{-n-l-3} + X_{(j-1)_l}^c(\eta')\eta'^{-n-2}]\right\}\\ &+ \sum_{s=0}^{l+n}\left(g_s^{l,n}\left\{nE_n\left[(n+l+1)J_{(j-1)_l}^c\eta'^{n+l-2} + (n-l)J_{(j-1)_{-l-1}}^c\eta'^{n-l-3} + \frac{1}{\eta'^2}\frac{\partial}{\partial\eta'}(X_{(j-1)_l}^c(\eta')\eta'^{n-1})\right]\right.\right.\\ &-(n+1)H_n\left[(l-n)J_{(j-1)_l}^c\eta'^{l-n-3} - (n+l+1)J_{(j-1)_{-l-1}}^c\eta'^{-n-l-4} + \frac{1}{\eta'^2}\frac{\partial}{\partial\eta'}(X_{(j-1)_l}^c(\eta')\eta'^{-n-2})\right]\\ &+\frac{nD_n}{2(2n+3)}\left[(n+l+3)J_{(j-1)_l}^c\eta'^{n+3} + (n-l+2)J_{(j-1)_{-l-1}}^c\eta'^{n-l-1} + \frac{1}{\eta'^2}\frac{\partial}{\partial\eta'}(X_{(j-1)_l}^c(\eta')\eta'^{n+1})\right]\\ &\left.+\frac{(n+1)G_n}{2(2n+1)}\left[(l-n+2)J_{(j-1)_l}^c\eta'^{-n+l-1} - (n+l-1)J_{(j-1)_{-l-1}}^c\eta'^{-n-l-2} + \frac{1}{\eta'^2}\frac{\partial}{\partial\eta'}(X_{(j-1)_l}^c(\eta')\eta'^{-n})\right]\right\}\\ &+ h_s^{l,n}\left\{E_n[J_{(j-1)_l}^c\eta'^{n+l-2} + J_{(j-1)_{-l-1}}^c\eta'^{n-l-1} + X_{(j-1)_l}^c(\eta')\eta'^{n+1}] + H_n[J_{(j-1)_l}^c\eta'^{l-n-3}\right.\\ &+ J_{(j-1)_{-l-1}}^c\eta'^{-n-l-4} + X_{(j-1)_l}^c(\eta')\eta'^{-n-3}] + \frac{(n+3)D_n}{2(n+1)(2n+3)}[J_{(j-1)_l}^c\eta'^{n+l} + J_{(j-1)_{-l-1}}^c\eta'^{n-l-1}\\ &\left.\left.\left.+ X_{(j-1)_l}^c(\eta')\eta'^{n+1}] - \frac{(n-2)G_n}{2n(2n-1)}[J_{(j-1)_l}^c\eta'^{-n+l-1} + J_{(j-1)_{-l-1}}^c\eta'^{-n-l-2} + X_{(j-1)_l}^c(\eta')\eta'^{-n-1}]\right\}\right)\right]\\ &\times g_m^c(\eta,\eta')S_m(\theta',\phi')\eta'^2\sin\theta'\,\mathrm{d}\theta'\,\mathrm{d}\phi'\,\mathrm{d}\eta'.\end{aligned} \tag{A3}$$

In expressions (A2) and (A3) the notation of Haber and Hetsroni [6] is adopted to define

$$S_k S_n \equiv \sum_i^{n+k} g_i^{n,k} S_i, \tag{A4}$$

$$\tilde{\nabla} . (S_k \tilde{\nabla} S_n) \equiv \sum_i^{n+k} h_i^{n,k} S_i, \tag{A5}$$

and

$$\tilde{\nabla} . (S_k \tilde{\nabla} \times \mathbf{t}_r S_n) \equiv \sum_i^{n+k-1} f_i^{n,k} S_i, \tag{A6}$$

with the nondimensional differential operator

$$\tilde{\nabla} \equiv \mathbf{t}_\theta \frac{\partial}{\partial \theta} + \mathbf{t}_\phi \frac{1}{\sin\theta} \frac{\partial}{\partial \phi}. \tag{A7}$$

APPENDIX B

I

For uniform flow parallel to the axis $\theta = 0, \theta = \pi$, in equations (A1) $n = 1$ and $p = 0$. The various coefficients in the expansions of the solid spherical harmonics are [16]

$$A_1^0 = 6\frac{U}{\Delta_1}\beta_1\gamma^{-2}(1-\gamma^5), \quad B_1^0 = -\frac{6}{10}\frac{U}{\Delta_1}\beta_1\gamma^{-2}(1-\gamma^5),$$

$$C_1^0 = 0, \quad D_1^0 = 6\frac{U}{\Delta_1}\gamma^3, \quad E_1^0 = -\frac{6}{10}\frac{U}{\Delta_1}\gamma^{-2}Y_1, \quad F_1^0 = 0,$$

$$G_1^0 = \frac{6}{10}\frac{U}{\Delta_1}\gamma^{-2}W_1, \quad H_1^0 = \frac{6}{10}\frac{U}{\Delta_1}\gamma^{-2}, \quad I_1^0 = 0,$$

with

$$\Delta_1 = \tfrac{6}{10}\gamma^{-2}(Y_1 - W_1\gamma),$$
$$Y_1 = 2(1+\beta_1) + 3\gamma^5(1-\tfrac{2}{3}\beta_1),$$
$$W_1 = 3(1+\tfrac{2}{3}\beta_1) + 2\gamma^5(1-\beta_1),$$

and

$$\beta_1 = \frac{\mu^c}{\mu^d + \tilde{\gamma}_1}.$$

Here, $\tilde{\gamma}_1$ is a parameter, termed the "interfacial retardation viscosity", which reflects the effects of adventitious surfactant impurities, adsorbed at the interface, on the rate of internal circulation within the particle. The functional dependence of this parameter on the mechanisms of surfactant adsorption–desorption and surface diffusion and convection is fully discussed in [14].

The velocity vector is nondimensionalized with respect to the uniform velocity U, and the Peclet number defined by $Pe = aU_s/\zeta$. Note, that the uniform velocity is related to the terminal sedimentation velocity U_s of a multi-particle assemblage [14] through

$$U_s = \tfrac{3}{2}\frac{U_s}{W_1}(Y_1 - W_1\gamma),$$

where

$$U_s = \frac{2}{9}\frac{a^2(\rho^d - \rho^c)(1-\gamma^3)\boldsymbol{g}}{\mu^c}.$$

For homogeneous shear (Couette) flow with respect to neutrally-buoyant particles, again with the direction of flow aligned with the axis $\theta = 0, \theta = \pi$, in equation (A1) $n = 2$ and $p = 2$. The various coefficients in the expansions of the solid spherical harmonics are [14]

$$\hat{A}_2^2 = \frac{15}{4}\frac{Ga}{\Delta_2}\beta_2\gamma^{-4}(1-\gamma^7), \quad \hat{B}_2^2 = -\frac{15}{56}\frac{Ga}{\Delta_2}\beta_2\gamma^{-4}(1-\gamma^7),$$

$$\hat{C}_2^2 = 0, \quad \hat{D}_2^2 = -\frac{15}{4}\frac{Ga}{\Delta_2}\gamma^3, \quad \hat{E}_2^2 = -\frac{5}{56}\frac{Ga}{\Delta_2}\gamma^{-4}Y_2, \quad \hat{F}_2^2 = 0,$$

$$\hat{G}_2^2 = -\frac{5}{14}\frac{Ga}{\Delta_2}\gamma^{-4}W_2, \quad \hat{H}_2^2 = -\frac{5}{28}\frac{Ga}{\Delta_2}\gamma^{-4}, \quad \hat{I}_2^2 = 0,$$

with

$$\Delta_2 = \frac{15}{14}\gamma^{-4}(Y_2 - W_2\gamma^3),$$
$$W_2 = 5(1+\tfrac{2}{5}\beta_2) + 2\gamma^7(1-\beta_2),$$
$$Y_2 = 2(1+\beta_2) + 5\gamma^7(1-\tfrac{2}{5}\beta_2),$$

and

$$\beta_2 = \frac{\mu^c}{\mu^d + \tilde{\gamma}_2}.$$

The "interfacial retardation viscosity" $\tilde{\gamma}_2$ is again defined in [14]. Note, that the velocity vector is now nondimensionalized with respect to Ga, and the Peclet number is accordingly defined $Pe \equiv Ga^2/\zeta$.

II

$$J^c_{(0)_1} = \tfrac{1}{2}(\psi_2 - \psi_1)\frac{1}{Q_1}\left(2m\frac{\xi^c}{\xi^d} + 1\right)\gamma, \tag{B1}$$

$$J^c_{(0)_{-2}} = \tfrac{1}{2}(\psi_2 - \psi_1)\frac{1}{Q_1}\left(m\frac{\xi^c}{\xi^d} - 1\right)\gamma, \tag{B2}$$

$$J^d_{(0)_1} = \tfrac{1}{2}(\psi_2 - \psi_1)\frac{3}{Q_1}\frac{\xi^c}{\xi^d}\gamma, \tag{B3}$$

$$Q_1 = \left(2m\frac{\xi^c}{\xi^d} + 1\right) + \left(m\frac{\xi^c}{\xi^d} - 1\right)\gamma^3, \tag{B4}$$

$$J^c_{(0)_0} = \tfrac{1}{2}(\psi_2 + \psi_1), \tag{B5}$$

$$J^d_{(0)_0} = \frac{1}{2m}(\psi_2 + \psi_1), \tag{B6}$$

$$J^c_{(1)_2} = \frac{1}{Q_2}\left[m\gamma^2(3T_{(1)_2} + V_{(1)_2}\gamma^3) + \frac{2\zeta^d}{\zeta^c}\gamma^2(T_{(1)_2} - U_{(1)_2}\gamma^3)\right], \tag{B7}$$

$$J^c_{(1)_{-3}} = \frac{1}{Q_2}\left[m(2T_{(1)_2}\gamma^2 - V_{(1)_2}) + \frac{2\zeta^d}{\zeta^c}(U_{(1)_2} - T_{(1)_2}\gamma^2)\right], \tag{B8}$$

$$J^d_{(1)_2} = \frac{\xi^d}{\xi^c}\frac{1}{Q_2}[V_{(1)_2}(\gamma^5 - 1) - U_{(1)_2}(2\gamma^5 + 3) + 5T_{(1)_2}\gamma^2], \tag{B9}$$

$$Q_2 = m(2\gamma^5 + 3) + \frac{2\zeta^d}{\zeta^c}(1 - \gamma^5), \tag{B10}$$

$$J^c_{(1)_0} = T_{(1)_0} + V_{(1)_0}\gamma, \tag{B11}$$

$$J^c_{(1)_{-1}} = -V_{(1)_0}, \tag{B12}$$

$$J^d_{(1)_0} = \frac{\xi^d}{\xi^c}\frac{1}{m}[T_{(1)_0} - U_{(1)_0} + V_{(1)_0}(\gamma - 1)], \tag{B13}$$

$$X^c_{(1)_2} = \frac{1}{W_1}\left[J^c_{(0)_1}\left(\frac{\eta^{-1}}{4} - \frac{W_1\eta}{8} - \frac{\gamma^5\eta^4}{14}\right) + J^c_{(0)_{-2}}\left(\gamma^5\eta - \frac{Y_1\eta^{-1}}{2} + \frac{5W_1\eta^{-2}}{8} + \frac{\eta^{-4}}{4}\right)\right], \tag{B14}$$

$$X^c_{(1)_0} = \frac{1}{W_1}\left[J^c_{(0)_1}\left(\frac{\eta^{-1}}{8} - \frac{\gamma^5\eta^4}{8} - \frac{Y_1\eta^2}{4} + \frac{W_1\eta}{2}\right) + J^c_{(0)_{-2}}\left(\frac{\gamma^5\eta}{2} - \frac{Y_1\eta^{-4}}{4} - \frac{W_1\eta^{-2}}{4} + \frac{\eta^{-4}}{8}\right)\right], \tag{B15}$$

$$X^d_{(1)_2} = \frac{1}{W_1}J^d_{(0)_1}\frac{1}{14}\eta^4\beta_1(1-\gamma^5), \tag{B16}$$

$$X^d_{(1)_0} = \frac{1}{W_1}J^d_{(0)_1}\tfrac{1}{4}\eta^2\beta_1(1-\gamma^5)(1-\tfrac{1}{2}\eta^2), \tag{B17}$$

$$T_{(1)_2} = \frac{1}{W_1}\left[J^c_{(0)_1}\left(\frac{W_1\gamma^{-1}}{2}-\frac{5\gamma}{28}\right)\right.$$

$$\left.-J^c_{(0)_{-2}}\left(\frac{5\gamma^4}{4}-\frac{Y_1\gamma}{2}+\frac{5W_1\gamma^2}{8}\right)\right], \quad \text{(B18)}$$

$$U_{(1)_2} = \frac{1}{W_1}\left[m\frac{\zeta^c}{\zeta^d}J^d_{(0)_1}\frac{\beta_1}{14}(1-\gamma^5)+J^c_{(0)_1}\left(\frac{\gamma^5}{14}+\frac{W_1}{2}-\frac{1}{4}\right)\right.$$

$$\left.+J^c_{(0)_{-2}}\left(\frac{Y_1}{2}-\gamma^5-\frac{5W_1}{8}-\frac{1}{4}\right)\right], \quad \text{(B19)}$$

$$V_{(1)_2} = \frac{1}{W_1}\left[\frac{\zeta^d}{\zeta^c}\frac{\zeta^c}{\zeta^d}J^d_{(0)_1}\tfrac{2}{7}(1-\gamma^5)+J^c_{(0)_1}\left(\frac{2\gamma^5}{7}+\frac{W_1}{8}+\frac{1}{4}\right)\right.$$

$$\left.+J^c_{(0)_{-2}}\left(1+\frac{5W_1}{4}-\gamma^5-\frac{Y_1}{2}\right)\right], \quad \text{(B20)}$$

$$T_{(1)_0} = \frac{3}{2W_1}\left[J^c_{(0)_1}\left(\frac{Y_1\gamma^{-2}}{6}-\frac{2\gamma}{28}-\frac{3W_1\gamma^{-1}}{8}\right)\right.$$

$$\left.+\tfrac{3}{8}J^c_{(0)_{-2}}W_1\right], \quad \text{(B21)}$$

$$U_{(1)_0} = m\frac{\zeta^c}{\zeta^d}\left[\frac{5}{56W_1}J^d_{(0)_1}\beta_1(1-\gamma^5)\right]$$

$$+\frac{3}{2W_1}\left[J^c_{(0)_1}\left(\frac{Y_1}{6}-\frac{3\gamma^5}{28}-\frac{3W_1}{8}\right)+\tfrac{3}{8}J^c_{(0)_{-2}}W_1\right], \quad \text{(B22)}$$

$$V_{(1)_0} = \frac{\zeta^d}{\zeta^c}\frac{\zeta^c}{\zeta^d}\left[-\frac{1}{7W_1}J^d_{(0)_1}\beta_1(1-\gamma^5)\right]$$

$$-\frac{3}{2W_1}\left[J^c_{(0)_1}\left(\frac{3\gamma^5}{7}-\frac{Y_1}{3}+\frac{3W_1}{8}\right)+\tfrac{1}{4}J^c_{(0)_{-2}}W_1\right], \quad \text{(B23)}$$

$$J^c_{(1)_3} = \frac{1}{Q_3}\left[3T_{(1)_3}m\gamma^3(1-\gamma^7+\gamma^{11})-3U_{(1)_3}\frac{\zeta^d}{\zeta^c}\gamma^7\right], \quad \text{(B24)}$$

$$J^c_{(1)_{-4}} =$$

$$\frac{1}{Q_3}\left[(3T_{(1)_3}\gamma^3-V_{(1)_3})m-3(T_{(1)_3}\gamma^7-U_{(1)_3})\frac{\zeta^d}{\zeta^c}\right], \quad \text{(B25)}$$

$$J^d_{(1)_3} = \frac{\zeta^d}{\zeta^c}\frac{1}{Q_3}\left[(3T_{(1)_3}\gamma^3-V_{(1)_3})(1-\gamma^7)\right.$$

$$\left.+(T_{(1)_3}\gamma^7-U_{(1)_3})(4+3\gamma^7)\right], \quad \text{(B26)}$$

$$Q_3 = m(4+3\gamma^7)+3\frac{\zeta^d}{\zeta^c}(1-\gamma^7), \quad \text{(B27)}$$

$$T_{(1)_3} = -\frac{1}{30(Y_2-W_2\gamma^3)}$$

$$\times\left[J^c_{(0)_1}\left(\frac{W_2}{12}-\frac{4}{9}\right)+J^c_{(0)_{-2}}\left(\frac{Y_2}{4}-\frac{7}{3}\gamma^5-\frac{W_2\gamma^3}{3}\right)\right], \quad \text{(B28)}$$

$$U_{(1)_3} = \frac{1}{30(Y_2-W_2\gamma^3)}$$

$$\times\left[m\frac{\zeta^c}{\zeta^d}J^d_{(0)_1}\frac{\beta_2}{18}(\gamma^7-1)-J^c_{(0)_1}\left(\frac{\gamma^7}{18}+\frac{W_2}{12}-\frac{1}{2}\right)\right.$$

$$\left.-J^c_{(0)_{-2}}\left(\frac{Y_2}{4}-\frac{11\gamma^7}{6}-\frac{W_2}{3}-\frac{1}{2}\right)\right], \quad \text{(B29)}$$

$$V_{(1)_3} = \frac{1}{30(Y_2-W_2\gamma^3)}$$

$$\times\left[\frac{\zeta^d}{\zeta^c}\frac{\zeta^c}{\zeta^d}J^d_{(0)_1}\frac{5}{18}\beta_2(\gamma^7-1)-J^c_{(0)_1}\left(\frac{5\gamma^7}{18}+1\right)\right.$$

$$\left.-J^c_{(0)_{-2}}\left(W_2-\frac{11\gamma^7}{3}+\frac{5}{2}\right)\right]. \quad \text{(B30)}$$

SOLUTIONS PAR DEVELOPPEMENT EN SERIE DU TRANSFERT THERMIQUE OU MASSIQUE AUX FAIBLES NOMBRES DE PECLET ET DE REYNOLDS DANS LES SYSTEMES BIPHASIQUES PARTICULAIRES CONCENTRES

Résumé—Des solutions générales de transfert de chaleur ou de masse aux faibles nombres de Péclet dans des systèmes biphasiques particulaires concentrés, plongés dans un écoulement arbitraire de fluide de Stokes avec des conditions aux limites arbitraires sur les températures ou les concentrations, ont été obtenues en utilisant des développements en série de perturbation de la variable dépendante suivant les puissances du nombre de Péclet. On a obtenu des solutions valables uniformément, à tout ordre d'approximation dans le domaine considéré, à la restriction près imposée sur la valeur du nombre de Péclet par les dimensions de ce domaine. Les solutions ont été appliquées à des cas d'intérêt pratique. Pour un système particulaire plongé dans un écoulement uniforme avec un gradient de température linéaire dans la direction de l'écoulement, les résultats se réduisent à ceux de Maxwell pour la conductivité électrique effective d'un milieu composite. On a également obtenu des expressions particulières pour les flux interfaciaux et les nombres de Nusselt locaux et globaux. Les résultats font apparaitre les effets de la fraction volumique des particules et le retard à la circulation interne dû aux impuretés tensioactives.

REGULÄRE EXPANSIONSLÖSUNGEN FÜR WÄRME- ODER STOFFÜBERTRAGUNG IN KONZENTRIERTEN ZWEIPHASEN PARTIKULÄRSYSTEMEN BEI KLEINEN PÉCLET- UND REYNOLDS-ZAHLEN

Zusammenfassung—Für den Wärme- oder Stoffübergang bei kleinen Péclet-Zahlen in konzentrierten Zweiphasen Partikulärsystemen in beliebigen Stokes'schen Strömungen mit beliebigen Temperatur- oder Konzentrationsrandbedingungen wurden allgemeine Lösungen erhalten mit Hilfe der regulären Störungsexpansion der abhängigen Variablen in Potenzen der Péclet-Zahl. Einheitlich gültige Lösungen für alle Näherungsordnungen ergaben sich für den interessierenden Bereich mit einer Einschränkung für die Werte der Péclet-Zahl, die aus den Dimensionen des Bereichs herrührt. Die Lösungen wurden auf Fälle praktischen Interesses beschränkt. Für ein partikuläres System in einheitlicher Strömung mit einem linearen Temperaturgradienten in Strömungsrichtung reduzierten sich die Ergebnisse auf jene von Maxwell für die effektive elektrische Leitfähigkeit in einem zusammengesetzten Medium. Spezielle Beziehungen wurden auch ermittelt für örtliche und Gesamtflüsse und für Nusselt-Zahlen. Die Ergebnisse geben auch die Einflüsse des Partikel-Volumen-Anteils und die Verzögerung der inneren Zirkulation durch Oberflächenverunreinigungen wieder.

РЕГУЛЯРНЫЕ РАЗЛОЖЕНИЯ В ВИДЕ РЯДА ДЛЯ ТЕПЛО ИЛИ МАССООБМЕНА В КОНЦЕНТРИРОВАННЫХ ДВУХФАЗНЫХ СИСТЕМАХ С МАКРОЧАСТИЦАМИ ПРИ НИЗКИХ ЧИСЛАХ ПЕКЛЕ И РЕЙНОЛЬДСА

Аннотация — Общие решения для тепло или массообмена при малых числах Пекле в концентрированных двухфазных системах макрочастиц, погруженных в произвольные стоксовые потоки при произвольных граничных условиях для температуры или концентрации, найдены в результате использования регулярных разложений возмущения зависимой переменной по степеням числа Пекле.

Получены решения, равномерно пригодные для всех порядков аппроксимации в интересующей нас области при условии, что ограничение на значение числа Пекле определяется размерами этой области. Решения приспособлены для случаев, представляющих практический интерес.

В случае системы с макрочастицами, погруженными в однородный поток с линейным температурным градиентом в направлении потока, результаты сводятся к результатам, полученным Максвеллом для эффективной электропроводности сложной системы. Найдены также локальные и суммарные межфазные потоки и числа Нуссельта.

Результаты показывают влияние объемной доли частиц и замедление внутренней циркуляции, вызванное примесью поверхностно-активных веществ.